

Loschmidt echo and dynamical fidelity in periodically driven quantum systems

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Abstract – We study the dynamical fidelity $\mathcal{F}(t)$ and the Loschmidt echo $\mathcal{L}(t)$, following a periodic driving of the transverse magnetic field of a quantum Ising chain (back and forth across the quantum critical point) by calculating the overlap between the initial ground state and the state reached after n periods τ . We show that $[\log \mathcal{F}(n\tau)]/L$ (the logarithm of the fidelity per-site) reaches a steady value in the asymptotic limit $n \rightarrow \infty$, and we derive an exact analytical expression for this quantity. Remarkably, the steady state value of $[\log \mathcal{F}(n\tau \rightarrow \infty)]/L$ shows memory of non-trivial phase information which is instead hidden in the case of thermodynamic quantities; this conclusion, moreover, is not restricted to 1-dimensional models.

The dynamical evolution of closed quantum systems has been recently upgraded from a rather academic question to a very intense field of research, due to many experimental advances, notably cold atom systems [1] and femtosecond resolved spectroscopies [2]. Many studies, in particular, have focused on different non-equilibrium issues, particularly when the quantum system is driven out of equilibrium either by a sudden quench or by a slow adiabatic change of the Hamiltonian (the so-called quantum annealing *alias* adiabatic quantum computation [3–7]). In parallel, there have been numerous studies on the so-called Kibble-Zurek scaling [8–13] of the defect density generated in the final state reached by driving a system across a quantum critical point (QCP) [14]: For reviews, see Refs. [15–17]. Many works have also explored the connection between quantum phase transitions, quantum information [18] and quantum critical dynamics [15]. Two important measures which show distinct behaviour close to a QCP happen to be the Loschmidt echo [19,20] and the ground state quantum fidelity [21]. Especially the former has been studied extensively in recent years in connection to the dynamics of decoherence [22–24], and the work statistics [25,26].

Considerably less studied is the case of a *time-periodic* driving, in which a closed quantum system is periodically

driven back-and-forth across a QCP [27,28]. Very recently, it has been argued that periodically driven closed quantum systems possessing an absolutely continuous Floquet spectrum are likely to display a “periodic steady state” [29,30], i.e., a periodic long-time dynamics for most physical observables, an effect that has been explicitly demonstrated for an Ising chain with a periodically driven transverse field.

In this paper we consider the Loschmidt echo [19,20,31] upon a periodic driving of a closed quantum system. The Loschmidt echo [31] is defined as:

$$\mathcal{L}(t) \equiv \left| \langle \Psi_0 | \hat{U}_0(0, t) \hat{U}(t, 0) | \Psi_0 \rangle \right|^2 \quad (1)$$

where \hat{U}_0 and \hat{U} are unitary evolution operators associated to two different Hamiltonians $\hat{H}_0(t)$ and $\hat{H}(t)$, and $|\Psi_0\rangle$ is some initial state. In the particular case in which \hat{H}_0 is time independent and $|\Psi_0\rangle$ is its ground state, then the Loschmidt echo is equivalently expressed as:

$$\mathcal{L}(t) = \left| \langle \Psi_0 | \hat{U}(t, 0) | \Psi_0 \rangle \right|^2 = |\langle \Psi_0 | \Psi(t) \rangle|^2 \equiv \mathcal{F}(t), \quad (2)$$

where the right-hand side might be viewed as a dynamical *fidelity*, i.e., the squared overlap between the initial state and the time-evolved state $|\Psi(t)\rangle = \hat{U}(t, 0) |\Psi_0\rangle$, *alias* the

return probability to the initial state $|\Psi_0\rangle$, or the amplitude P_0 of the zero-work delta peak, $P_0\delta(W)$, in the work distribution $P(W)$ [25, 26]. We stress that this is different from the conventional ground-state fidelity [21] $|\langle\Psi_0|\Psi_1\rangle|^2$, $|\Psi_{0(1)}\rangle$ being the ground-states of the Hamiltonian with two different sets of parameters: If the system repeatedly crosses a QCP during the evolution, the defects generated in the process manifest themselves in the dynamical fidelity $\mathcal{F}(t)$.

The key question we will address is whether $\mathcal{F}(t)$, while not being a standard thermodynamical observable, would still tend to reach a large- t steady state value [29] when the Hamiltonian $\hat{H}(t)$ is time-periodic, $\hat{H}(t+\tau) = \hat{H}(t)$. We will show here that the answer is positive for the explicit case we have studied in detail, i.e., a periodically driven quantum Ising chain, and we believe that this result extends to all the cases in which a periodic steady state is found for ordinary observables [29]. Moreover, and this is the key result of our study, the off-diagonal matrix elements whose long-time average is responsible for the periodic steady state of thermodynamical observables play here, for the dynamical fidelity, an important role: the correct long-time result, for which we derive an exact analytical expression, does not follow from a decohered density matrix [12].

In the following, we will concentrate on a simple model where quasi-analytic information can be extracted on the quantities of interest, namely the quantum Ising chain in transverse field. The Hamiltonian of the system is:

$$\hat{H}(t) = -\frac{1}{2} \sum_{j=1}^L (J\sigma_j^z\sigma_{j+1}^z + h\sigma_j^x) - \frac{v(t)}{2} \sum_{j=1}^{L_S} \sigma_j^x. \quad (3)$$

Here, the $\sigma_j^{x,z}$ are spin-1/2 Pauli matrices at site j for a chain of length L with periodic boundary conditions (PBC) $\sigma_{L+1}^{x,z} = \sigma_1^{x,z}$, and J is the standard longitudinal Ising coupling ($J = 1$ in the following). As for the transverse field terms, involving σ_j^x , we allow a uniform piece, h , as well as a time-dependent one, $v(t)$, acting only on a subchain of length L_S . We will take the time-dependent term $v(t)$ to be periodic, with a single harmonic, i.e., $v(t) = A \cos(\omega_0 t + \varphi_0)$. In the equilibrium case with a homogeneous transverse field (i.e., $A = 0$) the model has two (mutually dual) gapped phases, a ferromagnetic one ($|h| < 1$), and a quantum paramagnetic one ($|h| > 1$) separated by QCPs at $|h_c| = 1$ at zero temperature. When $A > 0$, the transverse field starts to oscillate periodically for $t \geq 0$ around the uniform value h , in a region of size L_S . In the following we will concentrate on the case in which $h = h_c$ (i.e., we perturb around the critical Ising Hamiltonian) and $L_S = L$ (i.e., the periodic driving acts on the whole chain) for which translational invariance can be exploited. The translationally non-invariant case can be dealt with using similar techniques, as explained in Ref. [29, 30]. Several questions can be addressed when $L_S < L$, in particular the expected markedly different behaviour of the case in which

$L_S = \text{const}$ as $L \rightarrow \infty$ (perturbing a finite region inside a very large system, which will act as “reservoir”), from the case in which $L_S/L = \text{const}$ as $L \rightarrow \infty$ (perturbing an extensive region). The case of a single-site driving, $L_S = 1$, is evidently connected with previous decoherence studies, usually concerned about sudden quench cases [32]. Also, there exist questions regarding the statistics of the work [25, 26] performed during the driving. These issues treated in the future, while here we focus on the case $L_S = L$, $h = h_c$, $A = 1$ and $\varphi_0 = 0$.

We shall now proceed to discuss what happens when a periodic driving is performed for a certain number n of periods $\tau = 2\pi/\omega_0$, by looking at the echo/fidelity stroboscopically at time $t = n\tau$, in a wide range of frequencies ω_0 . Quan and Zurek [33] have performed a similar study following a linear quenching of the transverse field, forward and backward, crossing the critical point twice and studying the interference of the phases thus accumulated in the wave function, as reflected in the fidelity. With a periodic driving scheme we can study the dynamical fidelity following successive passages through the QCP and explore its behaviour as a function of n for the entire range of the driving frequency ω_0 , including the extreme adiabatic ($\omega_0 \rightarrow 0$) and high-frequency ($\omega_0 \rightarrow \infty$) limits. As we shall discuss below, in the limit $n \rightarrow \infty$, coherence effects show up prominently for small frequencies, whereas in the opposite limit $\omega_0 \rightarrow \infty$ there is a sort of partial loss of coherence, though the dynamics remains fully coherent.

By going to Jordan-Wigner spinless fermions [34] and transforming to k -space, one can rewrite $\hat{H}(t)$ as a sum of two-level systems (see Refs. [13, 29, 30] for details). Using the basis states $|0\rangle$ and $|k, -k\rangle$ for each mode k , where $|0\rangle$ ($|k, -k\rangle$) denotes state with no (two) fermion(s), respectively, the two-level Hamiltonian could be cast into the form $\hat{H}_k(t) = (-h(t) + \cos k) \hat{\sigma}_z + (\sin k) \hat{\sigma}_y$ with $\hat{\sigma}_{y,z}$ being standard Pauli matrices and $h(t) = 1 + \cos \omega_0 t$. The state of the system can hence be factorized as $|\Psi(t)\rangle = \prod_{k>0} |\psi_k(t)\rangle$ with $k = (2p+1)\pi/L$ and $p = 0, \dots, (L/2 - 1)$, as appropriate for fermionic antiperiodic boundary conditions (we assume L to be a multiple of 4). Moreover, using the Floquet theory [35, 36] we can re-express each k -th component of the state $|\psi_k(t)\rangle$ as [29]:

$$|\psi_k(t)\rangle = r_k^+ e^{-i\mu_k t} |\phi_k^+(t)\rangle + r_k^- e^{i\mu_k t} |\phi_k^-(t)\rangle, \quad (4)$$

where $|\phi_k^\pm(t)\rangle$ are the (time-periodic) Floquet modes and $\pm\mu_k$ the corresponding Floquet quasi-energies, while the overlap factors $r_k^\pm = \langle\phi_k^\pm(0)|\psi_k(0)\rangle$ carry information about the initial state. Since $|\psi_k(n\tau)\rangle = r_k^+ e^{-i\mu_k n\tau} |\phi_k^+(0)\rangle + r_k^- e^{i\mu_k n\tau} |\phi_k^-(0)\rangle$, one readily arrives at the following expression for the fidelity after n complete oscillations of the driving field:

$$\mathcal{F}(n\tau) = e^{\sum_{k>0} \log |z_n(k)|^2}, \quad (5)$$

where $z_n(k) = |r_k^+|^2 e^{-i\mu_k n\tau} + |r_k^-|^2 e^{i\mu_k n\tau}$. For very large L , transforming the sum over k into an integral, one easily

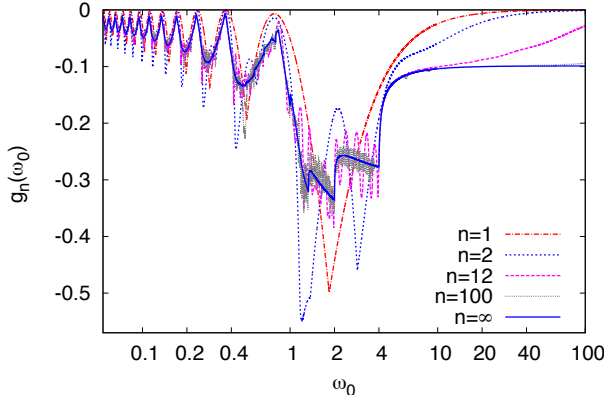


Fig. 1: $g_n = [\log \mathcal{F}(n\tau)]/L$, the logarithm of the fidelity per-site, as a function of the frequency ω_0 of the driving field $v(t) = \cos(\omega_0 t)$ for $n = 1, 2, 12, 100$. The value of g_∞ , Eq. (10), is also shown. Notice how, for any n finite, $g_n(\omega_0) \rightarrow 0$ for $\omega_0 \rightarrow \infty$ (for $n = 100$ such a rise towards 0 is visible right before $\omega_0 \sim 100$).

establishes that $\mathcal{F}(n\tau) \sim e^{Lg_n}$, where the logarithm of the fidelity (per-site) is well defined for $L \rightarrow \infty$ and given by:

$$g_n(\omega_0) = \int_0^\pi \frac{dk}{2\pi} \log |z_n(k)|^2. \quad (6)$$

All the Floquet-related quantities appearing in $z_n(k)$ (the overlaps r_k^\pm and the quasi-energies μ_k) are functions of the driving frequency ω_0 , a dependence that we have explicitly indicated only in $g_n(\omega_0)$. For $n = 0$, $|z_0(k)|^2 = (|r_k^+|^2 + |r_k^-|^2) = 1$, hence $g_0 = 0$ and $\mathcal{F}(0) = 1$, as expected. For $n > 0$, $|z_n(k)|^2 = |r_k^+|^4 + |r_k^-|^4 + 2|r_k^+|^2|r_k^-|^2 \cos(2\mu_k n\tau)$ and the phase-factors related to the Floquet quasi-energies μ_k will play a role.

Fig. 1 shows the behaviour of $g_n(\omega_0)$ versus the driving frequency ω_0 for a few values of n . Here $n = 1$ corresponds to a single oscillation of the field, i.e., a sweep across the QCP at $h_c = 1$, already showing clear interference effects between the two Kibble-Zurek crossings (back and forth) of the QCP. On the other hand, the case $n = 2$ shows more marked interference effects, while $n = 12$, and even more so $n = 100$, display a quite rich structure of peaks and dips as a function of ω_0 for $\omega_0 < 4$. (The value 4 marks, in our units, the upper limit of the natural frequencies of the unperturbed critical Ising model.) We notice, on the contrary, a rather smooth behaviour for $\omega_0 > 4$, with $g_n(\omega_0) \rightarrow 0$ for $\omega_0 \rightarrow \infty$: the larger the value of n , the larger will be the typical ω_0 beyond which $g_n(\omega_0)$ rises towards 0. Indeed, the limit $\omega_0 \rightarrow \infty$, for fixed finite n , involves a kind of *fast driving regime* in which the system “does not follow” the extremely fast oscillations of the field and “sees” the average Hamiltonian. This is not hard to justify from Eqs. (5),(6): In the limit $\omega_0 \rightarrow \infty$, the Floquet quasi-energies μ_k tend towards the unperturbed eigenvalues ϵ_k of the critical Ising model around which we are perturbing; since in that limit

$\tau = 2\pi/\omega_0 \rightarrow 0$, the phase-factors $e^{\pm i\mu_k n\tau} \rightarrow 1$ for every finite n , and therefore $|z_n(k)|^2 \rightarrow (|r_k^+|^2 + |r_k^-|^2) = 1$, which makes $\lim_{\omega_0 \rightarrow \infty} g_n(\omega_0) = 0$. On the contrary, the behaviour for $\omega_0 \leq 4$ is highly structured, with peaks and dips at various frequencies ω_0 . Notably, for the small values of ω_0 , there are sharp peaks where $g_n(\omega_0) \approx 0$ at all frequencies for which $J_0(2/\omega_0) = 0$, where J_0 is the zeroth order Bessel function; this is a quite common finding in this field, related to the phenomenon of “coherent destruction of tunnelling” [36,37]. We will address this point later, see Fig. 2 and accompanying discussion.

An obvious question arising from the inspection of Fig. 1 concerns what happens as $n \rightarrow \infty$: will g_n tend towards some definite limit, and if so, how can we calculate it? This question has to do with whether the system will effectively “synchronize” with the driving, i.e., if the stroboscopic observation of $g_n(\omega_0)$ will reveal a *steady state* reached for $n \rightarrow \infty$. The issue of reaching a “periodic steady state” when a closed quantum system is periodically driven has been raised and analyzed in detail, precisely for the model we are considering, in Ref. [29]. There, it was shown that extensive physical quantities — like the energy density, the density of defects generated, and the transverse magnetization — will indeed reach a periodic steady state predicted by the diagonal elements in the Floquet expansion, while off-diagonal terms will lead to a vanishing contribution for $n \rightarrow \infty$, due to destructive interference effects associated to the widely oscillating phase-factors $e^{\pm i\mu_k n\tau}$ when integrated over all momentum modes: mathematically, this is a consequence of the Riemann-Lesbesgue lemma predicting the vanishing, at large times, of Fourier transforms of sufficiently regular spectral densities. This can be rephrased by saying that the pure-state $\hat{\rho}_n = |\Psi(n\tau)\rangle\langle\Psi(n\tau)|$ can be effectively replaced, when calculating the average of most extensive physical quantities, with its *decohered* part [12]. In the Floquet basis, we would express this as follows:

$$\begin{aligned} \hat{\rho}_n &= \prod_{k>0} \begin{bmatrix} |r_k^+|^2 & r_k^+ r_k^{-*} e^{-2i\mu_k n\tau} \\ r_k^- r_k^{+*} e^{2i\mu_k n\tau} & |r_k^-|^2 \end{bmatrix} \xrightarrow{n \rightarrow \infty} \hat{\rho}_{\text{dec}} \\ \hat{\rho}_{\text{dec}} &= \prod_{k>0} \begin{bmatrix} |r_k^+|^2 & 0 \\ 0 & |r_k^-|^2 \end{bmatrix}. \end{aligned} \quad (7)$$

For the fidelity, however, the highly-oscillating phase factors due to the off-diagonal terms play a trickier and more important role. To better appreciate this, let us rewrite g_n as:

$$g_n(\omega_0) = \int_0^\pi \frac{dk}{2\pi} \log \frac{1 + q_k \cos(2\mu_k n\tau)}{1 + q_k}, \quad (8)$$

where $q_k \equiv 2|r_k^+|^2|r_k^-|^2/(|r_k^+|^4 + |r_k^-|^4) \in [0, 1]$. For very large n , the $\cos(2\mu_k n\tau)$ term oscillates rapidly as k runs over the BZ, and one would be tempted to conclude that $g_{n \rightarrow \infty} \rightarrow g_{\text{dec}}(\omega_0) = -\int_0^\pi \frac{dk}{2\pi} \log(1 + q_k)$, which is the result predicted by taking $g_{\text{dec}} = [\log \langle \Psi(0) | \hat{\rho}_{\text{dec}} | \Psi(0) \rangle]/L$. This conclusion, however, would be swift and wrong. We

could indeed write, using $\log(1+x) \leq x$, that:

$$\begin{aligned} g_n(\omega_0) &= g_{\text{dec}}(\omega_0) + \int_0^\pi \frac{dk}{2\pi} \log[1 + q_k \cos(2\mu_k n\tau)] \\ &\leq g_{\text{dec}}(\omega_0) + \int_0^\pi \frac{dk}{2\pi} q_k \cos(2\mu_k n\tau), \end{aligned} \quad (9)$$

where the integral on the right-hand side of the second expression tends to 0 for $n \rightarrow \infty$ (by the Riemann-Lesbesgue lemma), but this does not allow us to conclude that $\lim_{n \rightarrow \infty} g_n = g_{\text{dec}}$. The analysis of this limit is rather intricate in the general case. For $\omega_0 > 4$, however, things are much simpler, because q_k reaches its maximum value, $q_k = 1$, only at $k = 0$, where definitely $\cos(2\mu_k n\tau) \neq -1$ and hence there are no convergence issues in the expansion of the $\log[1 + q_k \cos(2\mu_k n\tau)]$. By expanding $\log[1 + q_k \cos(2\mu_k n\tau)]$, one can show that there are $\cos(2\mu_k n\tau)$ -related terms that survive when $n \rightarrow \infty$, originating from all even power terms of the expansion, for the simple reason that $\cos^{2p}(2\mu_k n\tau)$ (with p integer) does not average to zero even if n is very large and the oscillations are very fast. For $\omega_0 > 4$ we can derive the following closed analytical expression for g_∞ which, quite amusingly, fits our numerical data perfectly well even for $\omega_0 < 4$:

$$\begin{aligned} \lim_{n \rightarrow \infty} g_n(\omega_0) &= g_{\text{dec}}(\omega_0) - \sum_{p=1}^{\infty} \frac{(2p-1)!!}{2p(2p)!!} \int_0^\pi \frac{dk}{2\pi} q_k^{2p} \\ &= g_{\text{dec}}(\omega_0) - \int_0^\pi \frac{dk}{2\pi} \log \frac{2}{1 + \sqrt{1 - q_k^2}} \\ &= - \int_0^\pi \frac{dk}{2\pi} \log \frac{2(1 + q_k)}{1 + \sqrt{1 - q_k^2}}. \end{aligned} \quad (10)$$

Observe that, for large ω_0 , neither the q_k nor the Floquet states on which they depend show a dependence on ω_0 . This is a sign of the *fast driving regime*: when the driving is fast, the system is not able to follow the oscillations of the field and “sees” the average Hamiltonian. This leads to the high-frequency plateau visible in Fig. 1.

Even in the regime $\omega_0 < 4$, the overall structure of $g_n(\omega_0)$ for small/intermediate n is perfectly predicted by the analytical expression in Eq. (10). In the small- ω_0 regime, there are peaks of $g_n(\omega_0) \sim 0$ (small but finite negative values, really) at all the frequencies for which $J_0(2/\omega_0) = 0$. This result can be understood by looking at the modes close to the critical one at $k = 0$, for which $|k| \ll \omega_0$. Investigating the reduced two-level Hamiltonian in this limit (with a bias energy between the two states, appearing in the diagonal terms of the reduced Hamiltonian, $\epsilon_0 = 1 - \cos k \sim k^2/2$ for $k \rightarrow 0$, and the off-diagonal term $\sim k \ll \omega_0$), and assuming that the system is initially (at $t = 0$) in the state $|k, -k\rangle$, the squared amplitude of the state $|0\rangle$ at time t can be shown to be given by $\sin^2 \left[tk J_0\left(\frac{2}{\omega_0}\right) \right]$ (see Eq. (7) of Ref. [38]). Therefore, a coherent destruction of tunneling [36, 37] occurs for those values of ω_0 for which $J_0(2/\omega_0) = 0$: the system sticks

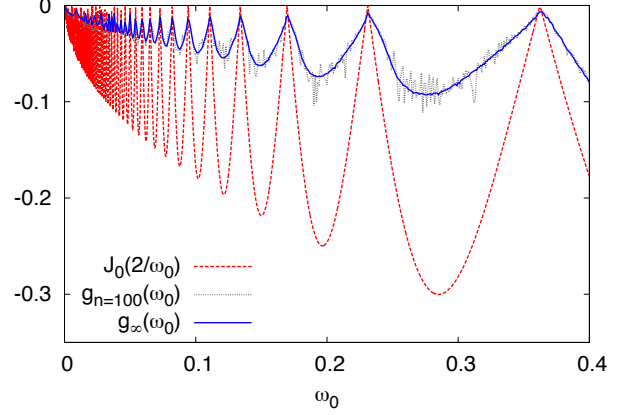


Fig. 2: Detail of the low-frequency region, showing that the peaks of $g_n(\omega_0)$ (where $g_n \approx 0$, but finite) closely match with the zeros of $|J_0(2/\omega_0)|$ for small ω_0 , all the way to the limit $n = \infty$. (The very small- ω_0 region suffers from numerical accuracy in sampling of the data.)

to its initial state, resulting in the peaks shown in Fig. 2. The regime $\omega_0 < 4$ also shows clear dips occurring for $\omega_0 = 4/m$ with $m = 1, 2, \dots$, due to quasi-degeneracies in the Floquet spectrum [29]. As shown in Fig. 3, the integrand of g_∞ , for instance, when analyzed versus k , shows clear negative sharp features at all k -points where the Floquet quasi-energies $\pm\mu_k$ become quasi-degenerate, either at 0 or at $\pm\omega_0/2$: these negative sharp features, in particular their crossing of the $k = \pi$ boundary at certain frequencies, are in the end responsible for the sharp dips observed in $g_n(\omega_0)$.

A crucial question is how much of this phenomenology depends on the one-dimensional nature of our system, and on the fact that the present model is integrable. It is difficult to firmly assess the role of integrability, since non-integrable systems are very difficult to tackle: as argued in Ref. [29], we believe however that the crucial ingredient is — rather than integrability — an absolutely continuous Floquet spectrum. Concerning the issue of dimensionality, it is simple to provide examples in higher dimensions where the same results can be found. Consider, for instance, a two-dimensional massive Dirac Hamiltonian $\hat{H}_D = \int d\mathbf{x} \hat{\Psi}^\dagger(\mathbf{x}) \left[m\hat{\sigma}_z - i\hbar v_F (\hat{\sigma}_x \partial_x + \hat{\sigma}_y \partial_y) \right] \hat{\Psi}(\mathbf{x})$, where $\hat{\Psi}(\mathbf{x})$ is a two-component spinor field operator describing, for instance, the effective low-energy degrees of freedom for electrons on a graphene lattice with unequal sublattice potentials around a single Dirac point with Fermi velocity v_F [39]. Variations of this model are ubiquitous in the field of topological insulators [40, 41]. The model has also been considered in the context of the ground state fidelity and Loschmidt echo in Ref. [42]. It shows a quantum phase transition when the mass m is tuned to zero. In momen-

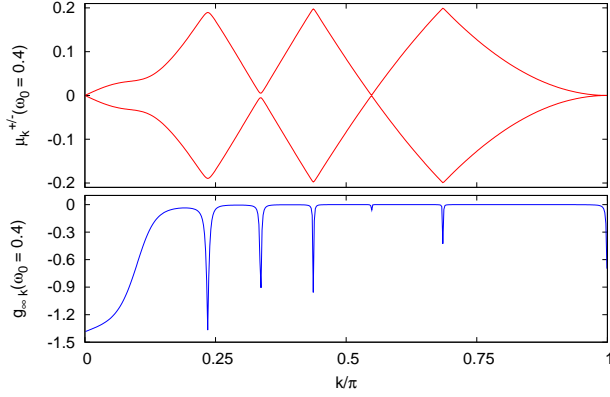


Fig. 3: The integrand of $g_\infty(\omega_0)$ in Eq. (10) (lower panel) versus k for $\omega_0 = 0.4$, together with the Floquet quasi-energies $\pm\mu_k$ (upper panel). The sharp negative features produce dips in g_∞ (and g_n , not shown) at $\omega_0 = 4/m$ with $m = 1, 2, \dots$ (here $m = 10$), when a resonance at $k = \pi$ is about to enter/leave the $k \in [0, \pi]$ integration region.

tum space, the Hamiltonian can be written as

$$\mathcal{H}_D(\mathbf{k}) = \begin{bmatrix} m & \hbar v_F(k_x - ik_y) \\ \hbar v_F(k_x + ik_y) & -m \end{bmatrix}, \quad (11)$$

with a momentum cutoff of $|k_{x,y}| \leq \pi/a$. Suppose the mass m is periodically driven across the QCP, $m(t) = m_0 \cos \omega_0 t$. The dynamical fidelity $\mathcal{F}(n\tau)$ can then be calculated by factorizing the state in terms of two-component spinor wavefunctions $|\psi_{\mathbf{k}}\rangle$ for each momentum, obtaining a result entirely similar to Eq. (8), i.e.:

$$g_n(\omega_0) = \int_0^{\frac{\pi}{a}} \frac{dk_x dk_y}{(2\pi)^2} \log \frac{1 + q_{k_x k_y} \cos(2\mu_{k_x k_y} n\tau)}{1 + q_{k_x k_y}}. \quad (12)$$

From this relationship, following an identical line of arguments, one can show that g_n indeed saturates, in the limit $n \rightarrow \infty$, to the analog of the analytic formula g_∞ in Eq. (10). An instance of this convergence is presented in Fig. (), where we show g_n vs n evaluated numerically for the Dirac model in Eq. (11) when $\omega_0 = 50$. We see that g_n indeed saturates to a steady state value g_∞ which is definitely below the value of g_{dec} obtained from the decohered density matrix. This picture holds true for all the frequencies we have checked, showing that also in this 2-dimensional case the asymptotic dynamical fidelity retains phase information which is lost in the thermodynamical observables.

In conclusion, we have studied the dynamical fidelity $\mathcal{F}(n\tau)$ of a quantum Ising chain following an n -period sinusoidal driving of the transverse field across the QCP. We addressed the question of whether $g_n = [\log \mathcal{F}(n\tau)]/L$ saturates to a well defined limit for large n , when the system reaches a periodic steady state as manifested in extensive thermodynamic quantities [29]. The answer is not really obvious: the presence of the logarithm in the expression of g_n forbids a direct application of the Riemann-Lesbesgue

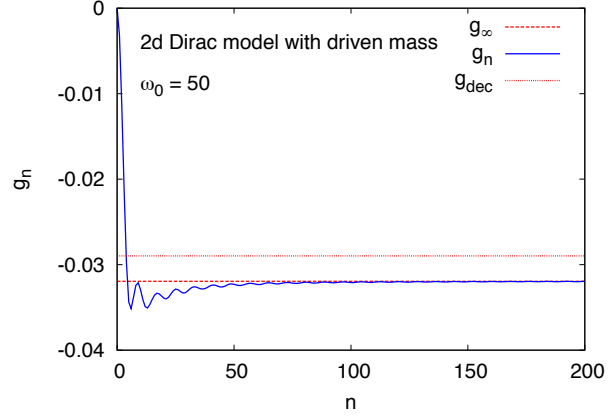


Fig. 4: $g_n = [\log \mathcal{F}(n\tau)]/L$, the logarithm of the fidelity per site, for a two-dimensional massive Dirac Hamiltonian (11) with a periodic mass driving $m(t) = m_0 \cos(\omega_0 t)$ versus n , for frequency $\omega_0 = 50$. The numerical calculation has been performed by discretizing the Brillouin zone momenta as $k_{x,y} = \frac{2\pi}{La} n_{x,y}$ with $L = 1000$ and $n_{x,y} = 0 \dots L - 1$. Observe that g_n quickly saturates to the analytical limit g_∞ for $n \rightarrow \infty$, well below the value g_{dec} obtained by using the decohered density matrix.

lemma and complicates the situation. Our results confirm that indeed g_n saturates to a steady value, and we were able to derive an exact expression for g_∞ which appears to work perfectly well for all values of ω_0 , yielding very detailed information on the exact position of peaks and dips of g_n even at intermediate values of n . However, the surprising result of our analysis is that g_∞ *cannot* be derived from the fully decohered mixed-state density matrix, $\hat{\rho}_{\text{dec}}$, where all phase information contained in the off-diagonal matrix elements is “effectively” lost, as instead possible for the thermodynamic quantities. Indeed, we have shown that g_n (and g_∞) is always strictly smaller than the result derived from $\hat{\rho}_{\text{dec}}$. In the high frequency region ($\omega_0 \gg 4$), we find that $g_\infty(\omega_0)$ saturates to a finite negative value independent of ω_0 showing that the system is unable to follow the oscillations of a too fast driving. With the example of a Dirac Hamiltonian in two dimensions, we have also shown that our main results are not limited to one-dimensional models.

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